

## List of Pre-Ph.D.Courses: Mathematics subject:

PAPER – I		Subject Code
S.No	Subject	
1	Real Analysis	PH24BM101
2	Complex Analysis	PH24BM102
3	Algebra	PH24BM103
4	Ordinary differential equations	PH24BM104
5	Partial differential equations	PH24BM105
6	Tensor calculus	PH24BM106
7	Probability & statistics	PH24BM107
8	Numerical analysis	PH24BM108
9	Number Theory	PH24BM109
10	Discrete Mathematics	PH24BM110

PAPER – II		Subject Code
S.No	Subject	
1	Topology	PH24BM201
2	Nonlinear Functional Analysis	PH24BM202
3	Fuzzy Algebra	PH24BM203
4	Mathematical modeling and chaotic dynamics	PH24BM204
5	Elements of Elasticity and Fluid Dynamics	PH24BM205
6	Boundary value problems	PH24BM206
7	Operations Research	PH24BM207
8	Relativity and cosmology	PH24BM208
9	Linear Algebra	PH24BM209
10	Boolean Algebra	PH24BM210

### Syllabus

#### UNIT-1-

#### (PH24BM101)Real Analysis

Basic Topology: Finite, Countable, and Uncountable Sets, Metric spaces, Compact sets, Connected sets.(Chapter 2 of the text book)

#### UNIT–II

Numerical Sequences and Series: Convergent sequences, Subsequences, Cauchy sequences, Upper and Lower limits, Some special sequences, Series, Series of non-negative terms, numbers, The Root and Ratio tests, Power series, Summation by parts, Absolute Convergence, Addition and Multiplication of series, Rearrangements.(Chapter3ofthe text book)

#### UNIT–III

Continuity: Limits of Functions, Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotone Functions, Infinite Limits and Limits at Infinity. (Chapter 4 of the text book)

#### UNIT–IV

Differentiation: The Derivative of a Real Function, Mean Value Theorems, The Continuity of Derivatives, L' Hospital's Rule, Derivatives of Higher order, Taylor's theorem, Differentiation of Vector- valued Functions. (Chapter 5 of the text book)

#### UNIT–V

Riemann-Stieltjes Integral: Definition and existence of the Riemann Stieltjes Integral, Properties of the Integral, Integration and Differentiation, the fundamental theorem of calculus – Integral of Vector- valued Functions, Rectifiable curves. (Chapter 6 of the prescribed text book)

TextBook: Principles of Mathematical Analysis by Walter Rudin, International Student Edition, 3rd Edition, 1985.

Reference: Mathematical Analysis by Tom M.Apostol, Narosa Publishing House, 2<sup>nd</sup> Edition,1985.

## **(PH24BM102)Complex Analysis**

### **UNIT-I**

Elementary properties and examples of analytic functions: Power series- Analytic functions- Analytic functions as mappings, Mobius transformations. (prescribed text book-1)

### **UNIT-II**

Complex Integration: Riemann - Stieltjes integrals - Power series representation of analytic functions- zeros of an analytic function -The index of a closed curve. (prescribed text book-1)

### **UNIT-III**

Cauchy's theorem and integral formula - the homotopic version of cauchy's theorem and simple connectivity- Counting zeros; the open mapping theorem. (prescribed text book-1)

### **UNIT-IV**

Singularities: Classifications of singularities- Residues- The argument principle. ( prescribed text book-1)

### **UNIT-V**

Conformal mapping-Bilinear Transforms-Argument Principle-Modulus theorem- Rouché's theorem (prescribed text book-2)

### **Prescribed text book:**

- 1.Functions of one complex variable by J.B.Conway : Second edition, Springer International student Edition, Narosa Publishing House, New Delhi.
- 2.Complex Variable and Applications by R.V. Churchill and J. W. Brown, Tata McGraw Hill, 2008.
- 3.Complex Analysis by L.V.Ahlfors, Tata McGraw Hill, 1979. 3. Foundation of Complex Analysis by S. Ponnuswamy,Narosa Publishing House, 2007.
- 4.Complex Variables: Theory and Applications by H.S. Kasana, PHI, 2006.

## **(PH24BM103)Algebra**

**UNIT I:**Normal Subgroups: Normal subgroups and Quotient groups - Isomorphism theorems – Auto morphisms -Conjugacy and G-Sets Cyclic Decomposition - Alternating group  $A_n$  – Simplicity of  $A_n$ .(Chapters 5 and 7) .

**UNIT II:** Direct Products - finitely generated abelian groups - Invariants of a finite abelian group - Sylow theorems - Groups of orders  $p^2$ ,  $pq$ . (Chapter 8).

**UNIT III:** Ideals, Homomorphisms, Sum and direct sum of ideals, Maximal and Prime Ideals.( Sections 10.1, 10.2, 10.3, 10.4 of Chapter 10)

**UNIT IV:** Nilpotent and Nil Ideals, Zorn's Lemma, unique factorization domains, Principal ideal domains, Euclidean domains, Polynomial rings over UFD (Sections 10.5 and 10.6 of Chapter 10 and Chapter 11).

**UNIT V:** Algebraic extension of fields: Irreducible polynomials and Eisenstein's criterion-Adjunction of roots-Algebraic extensions- Algebraically closed fields. (Chapter 15 – sections 15.1 – 15.4)

**Prescribed Book:** Basic Abstract Algebra: P. B. Bhattacharya, S. K. Jain and S. R. Nagapaul, Second edition, reprinted in India 1997, 2000, 2001.

**Reference Books:**

1. Topics in Algebra : [I. N. Herstein](#), 2<sup>nd</sup> Edition, John Wiley & Sons
2. Algebra : Thomas W. Hungerford, Springer
3. Algebra : Serge Lang, Revised Third Edition, Springer
4. Modern Algebra: Qazi Zameeruddin & Surjeet Singh , Eighth Edition, Vikas Publications

**(PH24BM104)ORDINARY DIFFERENTIAL EQUATIONS**

**UNIT-I:** Essential concepts from Real Function Theory – The basic problem -The fundamental existence and uniqueness theorem –examples to demonstrate the theory-continuation of solutions (Sections 10.1, 10.2 of the prescribed text book)

**UNIT-II:** Dependence of solutions on initial conditions – dependence of solutions on parameters (causal function f) - Existence and Uniqueness theorems for systems – existence and uniqueness theorems for Higher order equations – examples (Sections 10.3, 10.4 of the prescribed text book)

**UNIT-III:** Introduction to the theory of Linear differential systems – Theory and properties of Homogeneous linear systems (Sections 11.1 - 11.3 of the prescribed text book)

**UNIT-IV:** Theory of non-homogeneous linear systems – Theory and properties of the nth order homogeneous linear differential equations (Sections 11.4 - 11.6 of the prescribed text book)

**UNIT-V:** Theory of nth order Non homogeneous Linear equations – Sturm theory – Sturm Liouville Boundary value problems (Sections 11.7, 11.8, 12.1 of the prescribed text book)

**Prescribed Text Book:** Shepley L. Ross (2007). Differential Equations (3rd edition), Wiley India

**Reference book:** George F. Simmons (2017). Differential Equations with Applications and Historical Notes (3rd edition). CRC Press. Taylor & Francis.

**(PH24BM105)Partial Differential Equations**

**Unit I :**First Order Partial Differential Equations – Quasi linear PDEs – Pfaff's Equations (Sections 2.1, 2.2 of the prescribed text book)

**Unit II :**Nonlinear first order PDEs-Classification of the second order PDEs in two independent variables – wave, potential and Heat equations (Sections 2.3, 3.1 and 3.3 of the prescribed text book)

**Unit III :**Hyperbolic Equations – Cauchy problem for one dimensional wave equation – The Fourier method of Separation of variables (Sections 4.1, 4.3 of the prescribed text book)

**Unit IV :**Elliptic equations – Dirichlet problems involving Cartesian coordinates (Section 5.1 of the prescribed text book)

**Unit V :**Parabolic Equations – Cauchy problem – Mixed type problems (Sections 6.1, 6.2 of the prescribed text book)

Text book: Partial Differential Equations through Examples and Exercises, Endre Pap, Arpad Takaci and Djurdjica Takaci, Kluwer Texts in Mathematical Sciences, Volume 18, 1997 Springer Science+Business Media, Dordrecht

Reference: Elements of Partial Differential Equations, Ian Sneddon, McGraw-Hill International editions, New Delhi

### **(PH24BM106)Tensor calculus**

**Unit-1: Tensor Algebra:** Introduction, N-Dimensional space, Transformation of coordinates, Indicial and summation conventions, Contravariant vectors, Covariant vectors, Invariants, Second order tensors, Higher order tensors, Addition, subtraction, and multiplication of tensors, contraction, Quotient law, Conjugate symmetric tensors of the second order.

**UNIT-2 The Line Element:** Fundamental tensor, Length of a curve, Magnitude of a vector, Associate tensors, Angle between two vectors-Orthogonally, Principal directions.

**UNIT-3 Covariant Differentiation:** Christoffel symbols, Transformation law of Christoffel symbols, Covariant differentiation of vectors, Covariant differentiation of tensors, Laws of covariant differentiation, Intrinsic derivatives.

**UNIT-4 Geodesics-Parallelism:** Geodesics, Null-Geodesics, Geodesic coordinates, Parallelism, Covariant derivative.

**UNIT-5 Curvature Tensor:** Riemann-Christoffel tensor, Curvature tensor, Ricci tensor Curvature invariant, Bianchi's identity, Riemannian Curvature, Flat space, space of constant curvature. Cartesian Tensors: Orthogonal transformations, Rotations, Cartesian tensors, Infinitesimal strain, Stress, Equations of equilibrium, Generalized Hooke's law, isotropic tensors, Homogeneous and isotropic body, Curvilinear coordinates, Mechanics of continuous matter.

TEXT BOOK: Bary Spain, Tensor Calculus-Radha Publishing House, Calcutta

REFERENCE BOOKS: 1. Tensor Calculus, J.L.Synge and A.Schild, University of Toronto Press, Toronto.

2. Tensor Calculus, A.K.Agarwal, Krishna Prakasam mandir, Meerut.

3. Riemannian Geometry by L.P.Eisenhart, Princeton University Press, Princeton 1966

## **(PH24BM107)Probability & statistics**

**Unit-I** Random variables, distribution functions, Mathematical expectation and Generating functions: One and two dimensional random variables (Discrete and Continuous), Distribution functions, joint and conditional distribution functions, probability mass function, probability density function, Transformation of Random variables. Mathematical expectation, Moments of a distribution function, moment generating functions, characteristic functions and their properties, Chebychev inequality, probability generating functions. (Chapter 5, Chapter 6 except section 6.7 and Chapter 7-Sections 7.1, 7.2, 7.3, 7.5 and 7.9)

**Unit-II** Probability Distributions: Discrete Distributions-Binomial, Poisson and geometric distributions and their properties with applications. (Sections 8.1 -8.5 and 8.7 of Chapter8) Continuous distributions – Gamma, Beta, Cauchy, Normal distributions and their properties with applications (Sections 9.1, 9.2, 9.5, 9.6,9.7 and 9.12 of chapter 9)

**Unit-III** 1. Correlation and Regression: Correlation, Karl Pearson's coefficient of correlation, Calculation of correlation coefficient for Bivariate frequency distribution, Spearman's rank correlation coefficient. Linear regression- regression coefficients and their properties, angle between regression lines, standard error of estimate, curvilinear regression (Chapter 10 and Chapter 11)

**Unit-IV** Large Sample Theory :Types of sampling, tests of significance, procedure for testing of hypothesis, tests of significance for large samples, sampling of attributes, sampling of variables (Chapter 14)

**Unit-V** Exact Sampling Distributions: Exact sampling distributions,  $\chi^2$ , t, F distributions and their applications. (Chapter 15 up to 15.6.4 and Chapter 16 up to 16.6 except 16.4)

TEXT BOOK: Fundamentals of Mathematical Statistics-S.C.Gupta and V.K.Kapoor, 11 edition Sultan Chand and Sons, New Delhi

REFERENCE: An introduction to probability theory and mathematical statistics – V.K.Rohatgi Wiley Eastern Ltd, New Delhi

## **(PH24BM108)NUMERICAL ANALYSIS**

**Unit-I** Numerical techniques of solving transcendental and polynomial equations: Bisection methods, secant method, Newton-Raphson method, Chebyshev method, Rate of convergence. (Sec 2.1 - 2.5 of Textbook 1)

**Unit-II** Numerical techniques of solving system of Linear Algebraic equations: Triangularization method, Gauss elimination method, Gauss-Jordan method, Iterative methods: Jacobi method, Gauss-Seidel method. Numerical techniques of determining the eigen values and eigen vectors of a matrix: Jacobi method, Power method. (Sec 3.1-3.7, 3.11 of Text book 1).

**Unit-III** Approximation: Lagrange interpolation, Hermite interpolation, Spline interpolation, Least squares approximation. Methods based on undetermined coefficients – Gauss Legendre, Gauss Chebyshev integration methods, Lobatto integration, Composite integration

methods – Trapezoidal rule, Simpson's rule and Romberg integration. (Sec 4.1-4.6, 4.9, 5.6-5.10 of Text book.1).

**Unit-IV** Numerical techniques for solving ordinary differential equations: Euler method, backward Euler method, Midpoint method. Single step methods: Taylor series method, Runge-Kutta methods, Multistep methods: Predictor-corrector method, Adams Bashforth method, Adams –Moulton method, Convergence and stability analysis of single – step methods. (Sec 6.1-6.7 of Text book 1)

**Unit-V** Numerical methods for solving elliptic partial differential equations: Difference methods, Dirichlet problem, Laplace and Poisson equations. (Sec 1.1, 1.2, 4.1, 4.2 of Text book 2).

Text books: 1. Numerical method for Scientific and Engineering Computation, M.K. Jain, S.R.K. Iyengar and R.K. Jain, 6rd edition, 2012, New Age International Pvt. Ltd.

2. Computational methods for partial differential equations by M.K. Jain, S.R.K. Iyengar and R.K. Jain, New Age International Pvt. Ltd. (1993)

### (PH24BM109)Number Theory

#### **UNIT-I: ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION:**

Introduction- The Mobius function  $\mu(n)$  – The Euler totient function  $\phi(n)$ - A relation connecting  $\phi$  and  $\mu$  - A product formula for  $\phi(n)$ - The Dirichlet product of arithmetical functions- Dirichlet inverses and the Mobius inversion formula- The Mangoldt function  $\Lambda(n)$ -multiplicative functions- multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function-Liouville's function  $\lambda(n)$  - The divisor functions  $\lambda(n)$ -Generalized convolutions.

(Sections 2.1 to 2.14 of the Chapter 2 in the Prescribed Text Book.)

#### **UNIT-II: AVERAGES OF ARITHMETICAL FUNCTIONS:**

Introduction- The big oh notation. Asymptotic equality of functions- Euler's summation formula- Some elementary asymptotic formulas-The average order of  $d(n)$ - The average order of the divisor functions  $d(n)$  - The average order of  $n\sigma(n)$ -An application to distribution of lattice points visible from the origin. The average order of  $\mu(n)$  and  $\Lambda(n)$ . The partial sums of a Dirichlet product- Applications to  $\mu(n)$  and  $\Lambda(n)$ .

(Sections 3.1 to 3.12 of Chapter 3 in the Prescribed Text Book.)

#### **UNIT-III: SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMBERS:**

Introduction- Chebyshev's functions  $\psi(x)$  and  $\vartheta(x)$  - Relations connecting  $\vartheta(x)$  and  $\pi(x)$  - Some equivalent forms of the prime number theorem- Inequalities for  $\pi(n)$  and  $p_n$  - Shapiro's Tauberian theorem- Applications of Shapiro's theorem- An asymptotic formula for the partial sums

$\sum_{p \leq x} (1/p)$  - The partial sums of the Mobius function – The partial sums of the Mobius function.  $p \leq x$  Brief sketch of an elementary proof of prime number theorem.

#### **UNIT-IV: CONGRUENCES:**

Definition and basic properties of congruences- Residue classes and complete residue systems- Linear congruences- Reduced residue systems and the Euler- Fermat theorem-

Polynomial congruences modulo  $p$ . Lagrange's theorem- Applications of Lagrange's theorem- Simultaneous linear congruences. The Chinese remainder theorem- Applications of the Chinese remainder theorem.

(Sections 5.1 to 5.8 of the Chapter 5 in the Prescribed Text Book.)

#### **UNIT-V: FINITE ABELIAN GROUPS AND THEIR CHARACTERS:**

Characters of finite abelian groups- The character group- The orthogonality relations for characters- Dirichlet characters- Sums involving Dirichlet characters-The nonvanishing of  $L(1, \chi)$  for real non-principal  $\chi$

#### **DIRICHLET'S THEOREM FOR PRIMES IN ARITHMETIC PROGRESSION**

Introduction- Dirichlet's theorem for primes of the form  $4n-1$  and  $4n+1$ - The plan of the proof of Dirichlet's theorem

(Sections 6.5 to 6.10 & 7.1 to 7.3 of the Chapters 6 and 7 in the Prescribed Text Book.)

#### **Prescribed Text Book:**

Introduction to Analytic Number Theory, By T.M.APOSTOL-Springer Verlag-New York, Heidelberg-Berlin-1976.

#### **Reference Books:**

1. An Introduction to the theory of numbers, 5<sup>th</sup> edition by Ivan Niven Herbert S. Zuckerman and H. L. Montgomery, John Wiley & Sons INC. publications, U.K., 2008.
2. Elementary Number Theory, 7<sup>th</sup> edition by David M. Burton, 2011.

#### **(PH24BM110) Discrete Mathematics**

**Unit-I Mathematical logic:** Statements structures and notation, connectives, well formed formulas, tautologies, equivalences, implications, normal forms – Disjunctive and conjunctive, Principle disjunctive and conjunctive normal forms.

(Text book 1)

**Unit-II Theory of Inference:** Theory of inferences for statement calculus, validity using truth tables, rules of Inference. Predicate calculus: predicates, predicate formulas, quantifiers, free and bound variables, Inference theory of predicate calculus.

(Text book 1)

**Unit-III Relations and ordering: partially ordered relations,** Partially ordered sets, representation and associated terminology. Lattices, Lattices as partially ordered sets, some properties of Lattices, Lattices as algebraic systems, sub-Lattices, direct product and homomorphism some special Lattices. (Text book.1).

**Unit-IV Graph Theory:** Graphs and Multigraphs, Subgraphs, Isomorphism and Homomorphism, Paths, Connectivity, Traversable Multigraph, Labeled and Weighted Graphs, Complete, Regular and Bipartite Graphs, Trees, Planar Graphs. (textbook 2).

**Unit-V Directed Graphs:** Rooted Trees, Sequential Representation of Directed Graphs, Warshall's Algorithm, Shortest Path, Binary Trees, Complete and Extended

Binary Trees, Representation of Binary Trees, Traversing Binary Trees and Binary Search Trees (textbook 2).

Text books: 1. Discrete Mathematical structures with Applications to Computer Science by J.P. Trembly and R. Manohar, Tata Mc.Graw hill.

2. Discrete Mathematics, Schaum's outline series, second edition, by Seymour Lipschutz and Marc Lipson Tata Mc Graw-Hill.

### **(PH24BM201)Topology**

**UNIT-I:** Sets and Functions: Sets and Set inclusion – The algebra of sets – Functions – Products of sets – Partitions and equivalence relations – Countable sets – Uncountable sets – Partially ordered sets and lattices. (Chapter I: Sections 1 to 8 of the prescribed text book).

**UNIT-II;** Metric spaces: The definition and some examples – Open sets – Closed sets – Convergence, Completeness and Baire's theorem .  
(Chapter 2: Sections 9 to 12 of the prescribed text book).

**UNIT-III:** Metric spaces (Continued): Continuous mappings, Spaces of continuous functions – Euclidean and Unitary spaces.(Chapter 2: Sections 13 to 15 of the prescribed text book)  
Topological spaces: The definition and some examples – Elementary concepts– (Chapter 3: Sections 16 to 17 of the prescribed text book).

**UNIT-IV:** Topological spaces (continued): Open bases and open sub bases, Weak Topologies, The function algebras  $C(X, \mathbb{R})$  and  $C(X, \mathbb{C})$ . (Chapter 3: Sections 18 to 20 of the prescribed text book).  
Compactness: Compact spaces – Heine – Borel theorem (Chapter 4: Section 21).

**UNIT-V:** Compactness (continued): Product of Spaces – Tychonoff's theorem and locally Compact spaces – Compactness for metric spaces – Ascoli's theorem. (Chapter 4: Sections 22 to 25 of the prescribed text book).

**Prescribed Text Book:** Introduction to Topology and Modern Analysis by G. F. Simmons International Student edition – McGraw – Hill Kogakusha, Ltd.

### **(PH24BM202) Nonlinear Functional Analysis**

**UNIT-I:** Various Forms of Continuity, Geometry in Normed Spaces and Duality Mapping. Chapter 1, Sections 1.1 to 1.2 of the Text Book.

**UNIT –II:** Gateaux and Fréchet derivative, Properties of derivative, Taylor's theorem, Inverse function theorem and Implicit function theorem, Subdifferential of convex functions. Chapter 2 of the Text Book.

**UNIT –III:** Banach's contraction principle and its generalization. Chapter 4, Section 4.1 of the Text Book.

**UNIT-IV:** Non expansive mappings.  
Chapter 4, Section 4.2 of the Text Book.



**UNIT-V:** Fixed Point Theorems of Brouwer and Schauder.  
Chapter 4, Section 4.3 of the Text Book.

**Text Book:** Mohan C. Joshi and Ramendra K. Bose, Some Topics in Nonlinear Functional Analysis, Wiley Eastern Limited, Hyderabad, 1985.

**Reference:** V.I. Istratescu, Fixed Point Theory-An Introduction, Springer, 1981.

### **(PH24BM203)Fuzzy Algebra**

**UNIT-I:** From Classical (Crisp) sets to Fuzzy sets:- Introduction-Crisp sets: An overview-fuzzy set: Basic types-Fuzzy sets. Basic Concepts- Characteristics and significance of the paradigm shift (CH-1 of (I)). Fuzzy sets versus Crisp sets-Additional Properties of a cuts-Representations of Fuzzy setsExtension principle for Fuzzy sets (CH-2 of (I)).

**UNIT-II:** Operations on Fuzzy sets - Types of Operations - Fuzzy Compliments - Fuzzy Inter sections: tnorms - Fuzzy unions; t-Conorms - Combinations of operations - Aggregation Operations (CH-3 of (I)).

**UNIT-III:** Fuzzy Arithmetic -Fuzzy Numbers - Linguistic variables - Arithmetic operations on intervals - Arithmetic operations on Fuzzy numbers - Lattice of fuzzy numbers - Fuzzy equations (CH-4 of (I)).

**UNIT-IV:** Fuzzy Relations - Crisp versus fuzzy relations - Projections and Cylindric Extensions - Binary Fuzzy Relations - Binary Relations on a Single set - Fuzzy Equivalence Relations . (Sections 1 to 5, CH-5 of (I)).

**UNIT-V:** Fuzzy Compatibility Relations - Fuzzy Ordering Relations - Fuzzy Morphisms - Sup – i Compositions of Fuzzy Relations - Inf -wi Compositions of fuzzy Relations. (Sections 6 to 10, CH-5 of (I)).

**Prescribed Text Book:** (1) G.J.KLIR and BO YUAN, "Fuzzy sets and Fuzzy Logic, Theory and Applications" Prentice - Hall of India Pvt. Ltd., New Delhi., 2008.

### **(PH24BM204)Mathematical Modelling and Chaotic Dynamics**

**UNIT-I-**Introduction to Mathematical Modeling Introduction Introduction, Characteristics of Mathematical Models, Classification of Mathematical Models, Classification as per the Nature of Basic Equations, Some Simple Examples of Optimization Problems, Limitations Associated with Mathematical Modeling Modeling Approaches: Empirical Approach, Theoretical Approach, Stochastic or Probabilistic Approach, Deterministic Approach, Statistical Approach, Simulation Approach, Discrete and Continuous Approaches Modeling/Cyclic Processes , A Modeling Diagram , Compartment Models ,Mathematical Preliminaries, Construction of the Lyapunov Function and Testing of Stability Krasovskii's Method , Lyapunov Function for Linear Systems with Constant Coefficients, The Routh–Hurwitz Criterion for Stability, Stability Discussion Based on the Linearization Procedure, Global Asymptotic Stability , Limit Cycles , Liénard's Equation and Existence of a Limit Cycle, Energy Balance Method for Limit Cycles , Focus , Dynamic System and Its Mathematical Model , Hamiltonian Systems

**UNIT-II-**Modeling of Systems from Natural Science Introduction, Models with Single Population Continuous Time Models: Malthusian Model, Verhulst–Pearl Logistic Model, Gompertz Growth Model, Theta-Logistic Model, Model with Allee Effect, Limited Growth Model, Harvest Model, Models with Delay, Discrete Time Models, Linear Map

**UNIT-III-**Two-Dimensional (2D) Continuous Models (Modeling of Population Dynamics of Two Interacting Species) Analytical Tool, Kolmogorov Theorem, Local Stability Analysis, Lotka–Volterra Model, Variation of the Classical LV Model, Leslie–Gower Model, Rosenzweig–MacArthur Model, Variations of the RM Model, Prey–Generalist Predator Model, Holling–Tanner Model, Modified HT Model, Competition Model, Gause Model, 2D Discrete Models: Nicholson–Bailey Model, Modified NB Model, Aihara Model

**UNIT-IV-**Introduction to Chaotic Dynamics Introduction. Chaos and Chaotic Dynamics, Basin of Attraction, Primary Routes to Study Chaos, Types of Chaos, Transients, and Attractors, Methods of Investigation for Detecting Chaos, Method for Selection of Parameter Values, Calculation of the Basin Boundary Structures , 2D Parameter Scans,

**UNIT V :** Bifurcation Analysis

Bifurcation Diagrams, Hopf Bifurcation Analysis, Time-Series Analysis and Phase-Space Diagram Types of Bifurcations: Saddle-Node Bifurcation or Tangent Bifurcation, Transcritical Bifurcation, Pitchfork Bifurcation, Period-Doubling Bifurcation, Andronov–Hopf Bifurcation (Cycle Birth Bifurcation), Poincaré Map and Poincaré Section, Lyapunov Exponents

Text Book

1. Introduction to Mathematical Modelling and Chaotic Dynamics  
by Ranjit Kumar Upadhyay, Satteluri R.K. Iyengar, CRC Press
2. Mark Kot, 2001, Elements of Mathematical Ecology, Cambridge University Press

### **(PH24BM205)Elements of Elasticity and Fluid Dynamics**

**Unit-I** Kinematics of fluids, real and ideal fluids, velocity of fluid at a point, streamlines and path lines, velocity potential, velocity vector, local and particle rates of change, equation of continuity, Acceleration of fluid, conditions at a rigid boundary, General analysis of fluid motion (Chapter 2 of Text book 1)

**Unit-II** Equation of motion of a fluid, pressure at a point in a fluid at rest and in a moving fluid, conditions at a boundary of two in viscid immiscible fluids, Euler’s equations of motion, Bernoulli’s equation. Discussion of the case of steady motion under conservative body forces, Vortex motion, Kelvin’s circulation theorem. Some further aspects of vortex motion (Chapter 3(excluding sections 3.8 to 3.11) of Text book 1).

**Unit-III** Some two - dimensional flows: Meaning of two - dimensional flow, use of cylindrical polar coordinates, the stream function, the complex potential for two – dimensional, irrotational, incompressible flow, complex potential for standard two – dimensional flows, some worked examples, two - dimensional image systems. The Milne-

Thomson circle theorem, the theorem of Blasius (Chapter 5(excluding sections 5.10 to 5.12) of Text book 1).

**Unit-IV** Analysis of strain: Deformation, affine deformation, infinitesimal affine deformation, geometrical interpretation of the components of strain, strain quadric of Cauchy, principal directions, invariants, general infinitesimal deformation, Examples of strain, equations of compatibility, finite deformations. (Chapter 1 of Text book 2)

**Unit-V** Analysis of stress, body and surface forces, stress tensor, equations of equilibrium, transformation of coordinates, stress quadric of Cauchy, Mohr's diagram, examples of stress (Chapter 2 of Text book2)

Text books: 1. Text book of Fluid Dynamics by F.Chorlton, CBS publishers and distributors, New Delhi.

2. Mathematical theory of Elasticity, by I.S.SOKOLNIKOFF 2 nd edition; Tata Mc Graw Hill-New Delhi

### **(PH24BM206)BOUNDARY VALUE PROBLEMS**

**UNIT – I** General theory for linear first order system of differential equations, Existence of solutions, Solution space. The first order non-homogeneous equation, variation of parameters. The adjoint nth order equation. Relation between scalar and vector adjoints.

**UNIT – II** The two point boundary value problems, Homogeneous two-point boundary value problems, the adjoint boundary problem, the non-homogeneous boundary problem, Green's matrix and self – adjoint boundary value problem.

**UNIT – III** Introduction to Eigen value problems, the vibrating string problem, Heat conduction problem, properties of the Green's operator. Existence of Eigen values and Eigen functions.

**UNIT IV** Non – linear boundary value problems, kinds of boundary value problems, the Generalized Lipschitz condition, failure of existence and uniqueness to Linear boundary value problem, relation between first and second boundary value problems. A more general Lipschitz condition, application to boundary value problems (Chapters 1,2, and 3 of Ref. 4).

**UNIT – V** Stability: Definition and examples Liapunov method for uniform stability, Asymptotic stability. Linear and quasi-linear ordinary differential systems, Autonomous Ordinary differential systems, trajectories and critical points, linear systems of second order, critical points of quasi-linear systems of second order.

Books: 1. Theory of Ordinary and delay differential equations by R.D. Driver Kingston R.I., Nov, 1976(Springs Verlag)

2. Theory of ordinary differential equations by E.A. coddington and N. Levinson.

3. Theory of ordinary differential equations by R.H. Cole, appleon century – crofts, New York, 1968. 4. Non-Linear two point boundary value Problems by P.B. Bailey, L.F. Shampine and P.E. Waltman, Academic press, New York, London (1968)

## **(PH24BM207)Operations Research**

**UNIT-I: Linear Programming: Simplex Method:** Introduction-Fundamental properties of solutions-The computational procedure-Use of artificial variables.  
(Sections 4.1 to 4.4 of the Chapter 4 in the Prescribed Text Book.)

**UNIT-II: Duality in Linear Programing:** Introduction-General Primal-Dual pair-Formulating a Dual problem-Prime-Dual Pair in matrix form-Duality theorems-Complementary slackness theorem- Duality and simplex method.  
(Sections 5.1 to 5.7 of the Chapter 5 in the Prescribed Text Book.)

**UNIT-III: Duality in Linear Programing :** Economic Interpretation of Duality, Dual Simplex method **Post-optimal Analysis :** Introduction-Variation in the cost vector-Variation in the requirement vector-variation in the coefficient matrix-Structural variations-Applications of Post-optimal Analysis.  
(Sections 5.8, 5.9 and 6.1 to 6.6 of the Chapters 5 and 6 in the Text Prescribed Book.)

**UNIT-IV: Transportation Problem and Assignment Problem:** Introduction-General transportation problem-The transportation table-Solution of a transportation problem-Finding an initial basic feasible solution-Test for optimality-Degeneracy in Transportation problem-Transportation Algorithm (MODI Method)- Introduction -Mathematical formulation of the problem-The Assignment method-Special cases in Assignment problem-A typical Assignment problem.  
12 hours (Sections 10.1 to 10.3 and 10.8 to 10.11 of the Chapter 10 in the Prescribed Text Book.) (Sections 11.1 to 11.5 of the Chapter 11 in the Prescribed Text Book.)

**UNIT-V: Games and Strategies:** Introduction-Two-person zero-sum games-some basic terms-The maximin-minimax principle-Games without saddle points-Mixed strategies-Graphic solution of  $2 \times n$  and  $m \times 2$  games.  
(Sections 17.1 to 17.6 of the Chapter 17 in the Prescribed Text Book.)

**Prescribed Text Book:** Operations Research, Kanti Swarup, P.K. Gupta and Man Mohan Sultan Chand & Sons, New Delhi, 2006.

### **Reference Books:**

1. Operations Research, An Introduction: Hamdy A Taha, Maxwell Macmillan International Edition, New York, 1992.
2. Operations Research Theory, methods and Applications, S.D. Sarma, kedar nath Ram nath publications, 2008.

## **(PH24BM208)RELATIVITY AND COSMOLOGY**

**Unit-I** Tensor Analysis: N-dimensional space, covariant and contravariant vectors, contraction, second & higher order tensors, quotient law, fundamental tensor, associate tensor, angle between the vectors, principal directions, Christoffel symbols, covariant and intrinsic derivatives, geodesics (Chapters 1 to 4 of Text book.1).

**Unit-II** Riemann Christoffel Tensor, covariant curvature tensor and its properties, Ricci Tensor, Curvature invariant, Einstein space, Bianchi's identity, Riemannian Curvature, Einstein space, flat space, space of constant curvature, Schur's Theorem (Chapter V of Text book.1).

**Unit-III** Space-time continuum, the three plus one dimensions of space-time, the geometry corresponding to space-time, the signature of the line element and the three kinds of interval, Lorentz rotation of axes, transformation to proper coordinates (Chapter II, Articles 13-18 of Text book 2).

**Unit-IV** The mass of a moving particle, the transformation equations for mass, work and kinetic energy, the relations between mass, energy and momentum, Four-dimensional expressions of the mechanics of a particle (Chapter III, Articles 23-28 of Text book 2).

**Unit-V** The Maxwell-Lorentz Field Equations, The transformation equations for E, H and Q. The force on a moving charge, The energy and momentum of electromagnetic field, electromagnetic stresses, Four dimensional expressions for electron theory (Chapter IV, Articles 39-43 & 46 of Text book 2).

Text Books:

1. Barry Spain, Tensor Calculus-Radha Publishing House, Calcutta.
2. R.C. Tolman, Relativity, Thermodynamics and Cosmology, Clarendon Press, Oxford.

Books

Reference:

1. Introduction to Special Relativity by Robert Resnick, John Wiley & Sons, New York.
2. Theory of Relativity by S.R. Roy and Raj bali Jaipur Publishing House, Jaipur.
3. J.K. Goyal and K.P. Gupta, Theory of Relativity, Krishna Prakasan Media(P) Ltd., Meerut

### **(PH24BM209)Linear Algebra**

**Unit – I:** Introduction, Characteristic Values, Similar Matrices, Diagonalizable Operators, Annihilating Polynomials, Minimal Polynomials, Cayley – Hamilton Theorem  
(Sections 6.1 - 6.3 of Chapter 6 in the Prescribed Text Book)

**Unit – II:** Invariant Subspaces, T-conductor of a vector, T-annihilator of a vector, Simultaneous Triangulation; Simultaneous Diagonalization.  
(Sections 6.4 - 6.5 of Chapter 6 in the Prescribed Text Book)

**Unit – III:** Direct – Sum Decompositions, Projections, Invariant Direct Sums, The Primary Decomposition Theorem.  
(Sections 6.6 – 6.8 of Chapter 6 in the Prescribed Text Book)

**Unit – IV:** Cyclic Subspaces and Annihilators, T-cyclic Subspace Generated by a Vector, Companion Matrices, Complementary Subspaces, I-admissible Subspaces, Cyclic Decompositions and Rational form, Generalized Cayley – Hamilton Theorem Invariant Factors.  
(Sections 7.1, 7.2 of Chapter 7 in the Prescribed Text Book).

**Unit – V:** The Jordan Forms, Elementary Jordan Matrix with Characteristic Value ,

Computation of Invariant Factors, Elementary Matrices, Smith Normal Forms, Summary;  
Semi-Simple Operators.

(Sections 7.3 – 7.5 in the Prescribed Text Book)

**Prescribed Text Book:** Linear Algebra by Kenneth Hoffman and Ray Kunze, Prentice-Hall India Pvt. Ltd, 2<sup>nd</sup> Edition, New Delhi.

### **(PH24BM210) Boolean Algebra**

**Unit-I :** Boolean rings – Boolean algebras – Fields of sets - Elementary relations-

**Unit-II :** Order – Infinite operations – Subalgebras – Homo morphisms-

**Unit-III :** Free Algebras- Ideals and filters – The Homo morphisms theorem-

Boolean o-algebras-

**Unit-IV :** The countable chain condition – Measure algebras - Atoms – Boolean spaces –  
The representation theorem -Duality for ideals

**Unit-V :** Duality for Homo morphisms. Boolean  $\sigma$  spaces, Represanation of  $\sigma$  algebra  
, Boolean Measure spaces.

Text Book: Lectures on Boolean Algebras, by Paul R. Halmos, D. Van Nostrand Company,  
Inc. Princeton, New Jersey